

Topics for the math and statistics exam
HSE University – Saint-Petersburg
Master program
"Data Analytics for Business and Economics"
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Description of the exam

Students will have one week to complete the online test and submit their answers. To qualify for the interview stage, a minimum score of 60% is required.

During the interview, students may be asked questions related to the exercises from the online test or to similar problems. Candidates are therefore encouraged to be prepared to explain their solutions step by step, to discuss the reasoning behind their answers, and to describe the mathematical and statistical concepts applied.

Please note that providing correct answers without being able to clearly explain the underlying reasoning will be considered academic dishonesty.

The test is offered in two versions: one for applicants to the Management track (Customer Analytics or People Analytics) and one for applicants to the Economics track.

The Economics track requires a solid command of both mathematics and statistics. The test for the Management track focuses more on statistical analysis, the correct application of econometric tools, and the interpretation of results. Nevertheless, the Management track test also requires to be familiar with the fundamentals of linear algebra, differentiation, integration, equation-solving techniques, and optimization with equality constraints in one or multiple variables.

An example of the test, as well as the list of recommended textbooks for preparation, is provided below.

Possible Topics for the Master's in Economics Entry Exam

Mathematics

- **Linear Algebra**

- Matrix and vector operations; identity and null matrices
- Transposes and inverses; finding the inverse matrix
- Determinants and Cramer's rule
- Conditions for nonsingularity of a matrix
- Rank of a matrix; diagonalization
- Systems of linear equations and inequalities
- Input-output models

- Eigenvalues and eigenvectors

- **Calculus and Analysis**

- Concept of a limit; rules of differentiation
- Exponential and logarithmic functions; interest compounding and the function Ae^n
- Partial differentiation; differentiation of inverse functions
- Gradient vector and Jacobian
- Mean value theorem; polynomial approximation
- Second and higher derivatives; Hessian matrix
- Taylor's theorem
- L'Hôpital's rule
- Change of variables in integration

- **Optimization and Comparative Statics**

- Relative vs. absolute extrema; first- and second-derivative tests
- Tests for concavity and convexity of multivariable functions
- Properties of convex functions; quasiconvexity and quasiconcavity
- Unconstrained optimization
- Optimization with equality and inequality constraints
- The Lagrangian function and Kuhn–Tucker conditions
- Constraint qualification; shadow prices and the value function
- The envelope theorem and maximum value function
- Utility maximization and consumer demand

- **Functions and Differential Equations**

- Homogeneous and linear functions
- Linear approximation and the derivative
- Systems of nonlinear equations
- The inverse and implicit function theorems
- First- and second-order differential equations
- The Bellman equation (dynamic optimization)

- **Integration and Related Topics**

- Indefinite, definite, and improper integrals
- Double integrals
- Present value and continuous compounding

Statistics

- **Descriptive Statistics**

- Types of data: cross-sectional, time series, panel
- Measures of central tendency: mean, median, mode
- Measures of dispersion: variance, standard deviation, coefficient of variation
- Covariance and correlation

- **Probability Theory**

- Random variables (discrete and continuous)
- Probability distributions: Normal, t , chi-square, F
- Joint, marginal, and conditional distributions
- Expectation, variance, and covariance
- Law of large numbers and central limit theorem

- **Estimation and Sampling**

- Random sampling and sampling distributions
- Point and interval estimation
- Properties of estimators: bias, consistency, efficiency, sufficiency
- Method of moments and maximum likelihood estimation (MLE)

- **Statistical Inference**

- Hypothesis testing: null and alternative hypotheses, Type I/II errors
- t -, F -, and chi-square tests
- p -values and confidence intervals
- One- and two-sample comparisons

- **Regression Analysis**

- Simple and multiple linear regression
- Model assumptions and estimation via OLS
- Multicollinearity, heteroskedasticity, and autocorrelation
- Goodness of fit: R^2 and adjusted R^2
- Gauss–Markov theorem and BLUE estimators
- Dummy variables and categorical data handling
- Nonlinear models and transformations (logs, polynomials)

- Statistical software output interpretation

- **Advanced and Applied Topics**

- Probability models: Logit and Probit
- Stationarity and autocorrelation (intro to time series)

Textbooks

- Michael Carter, *Foundations of Mathematical Economics*, MIT Press, 2001.
- A. de la Fuente, *Mathematical Methods and Models for Economists*, Cambridge University Press, 2000.
- Alpha C. Chiang and Kevin Wainwright, *Fundamental Methods of Mathematical Economics*, 4th Edition, McGraw-Hill, 2005.
- Carl P. Simon and Lawrence E. Blume, *Mathematics for Economists*, W. W. Norton & Company, 1994.
- Knut Sydsæter, Peter Hammond, Atle Seierstad, and Arne Strøm, *Further Mathematics for Economic Analysis*, Pearson, 2005.
- Knut Sydsæter, Peter Hammond, and Arne Strøm, *Essential Mathematics for Economic Analysis*, Prentice Hall, Inc., 2012.

Example of the test

1 Test for the Management track

Problem 1 In an effort to plan out expenses, the Roberts family is representing its annual budget as a circle graph. Each sector of the graph is proportional to the amount of the budget it represents. If “clothes and shoes” takes up 54° (54 degrees) of the chart, how much of the Roberts’s \$20,000 annual budget is dedicated to clothes and shoes?

Problem 2 Over a three-week period, the price of an ounce of gold increased by 25% in the first week, decreased by 20% in the following week, and increased by 5% in the third week. If the price of gold was 1000 dollars per ounce at the beginning of the three weeks, what was the price at the end of the three weeks?

Problem 3 The table below shows the enrollment in various classes at a certain college.

Class	Number of students
Biology	50
Physics	35
Calculus	40

Although no student is enrolled in all three classes, 15 are enrolled in both Biology and Physics, 10 are enrolled in both Biology and Calculus, and 12 are enrolled in both Physics and Calculus. How many different students are in the three classes?

Problem 4 In a nationwide poll, 1000 people were asked 2 questions. If 25 answered “yes” to question 1, and of those 13 also answered “yes” to question 2, which of the following represents the number of people polled who did not answer “yes” to both questions?

Problem 5 If a copier makes 3 copies every 4 seconds, then continues at this rate, how many minutes will it take to make 9,000 copies?

Problem 6 To be considered grade AA, an egg must weigh between 75 and 90 grams, including the shell. Shells of grade AA eggs weigh between 3 and 5 grams. What is the smallest possible mass, in grams, of a 12-egg omelet, assuming that only grade AA eggs are used, the shells are all discarded, and no mass is lost in the cooking process?

Problem 7 A group of 7 fishermen chartered a boat for a day to fish for flounder. The boat costs 1000 dollars per day to rent. If the group can find 3 more fishermen on the docks who are willing to come aboard and share the rental costs, how much less will the rental cost be per person in terms of x ?

Problem 8 If $3(x^2 + x) - 7 = x^2 + 2(4 + x^2)$, then what's the value of x ?

Problem 9 Suzie's Discount Footwear sells all pairs of shoes for one price and all pairs of boots for another price. On Monday the store sold 22 pairs of shoes and 16 pairs of boots for \$650. On Tuesday the store sold 8 pairs of shoes and 32 pairs of boots for \$760. How much more do pairs of boots cost than pairs of shoes at Suzie's Discount Footwear?

Problem 10 Solve

$$7(x - 3) - 2 = -3(x + 1)$$

Problem 11 Solve

$$x^2 - 3x + 2 = 0$$

. Write down the arithmetic average of the two roots.

Problem 12 Solve

$$\begin{cases} 2x + 3y = 7 \\ 3x - 2y = 4 \end{cases}$$

Write $x^2 + y$ as your answer.

Problem 13 Solve

$$\ln(x) + \ln(2x) = \ln(8)$$

for $x > 0$

Problem 14 Solve

$$(x - 2)(x - 1)\sqrt{\ln(x - 1)} = 0$$

Find the arithmetic average of the roots of this equation.

Problem 15 For matrices A, B, and C which of the following is not generally true

1. $A(B + C) = AB + AC$
2. $AB \neq BA$
3. $A(BC) = (AB)C$
4. $A(A + B) + B(A + B) = AA + 2AB + BB$

Problem 16 The graph of a linear function passes through the points $x_1 = (9, 9)$ and $x_2 = (14, 19)$. Find the second coordinate of the third point $x_3 = (5)$

Problem 17 Solve for x :

$$\frac{x - 4}{x - 1} \geq 2$$

What is the maximum value of x that it can attain? Write 777 if such value does not exist.

Problem 18 Determine the derivative $f'(x)$ of the function $f(x)$ below.

$$f(x) = 5 + \sqrt{x} + x^4$$

Calculate its value at point $x = 4$.

Problem 19 Determine the derivative $f'(x)$ of the function $f(x)$ below.

$$f(x) = (x^3 + 2x + 1)e^{2x+1}$$

Calculate its value at point $x = -1$.

Problem 20 Determine the derivative $f'(x)$ of the function $f(x)$ below.

$$f(x) = (x^4 + 4x^2 + 1)^3$$

Calculate its value at point $x = 2$.

Problem 21 Your deposit in a bank at this moment is 15,000 euro. The interest rate is 2.4 % per year. How much should you have deposited in a bank 10 years ago to have this deposit now?

Problem 22 Your deposit in a bank at this moment is 15,000 euro. The interest rate changes to 2.55 % per year. Calculate the value of your deposit 5 years from now.

Problem 23 You buy a device for 25,500 euro. It depreciates 12 % per year. What is its value 5 years later?

Problem 24 At which point does the following function attain its local maximum?

$$f(x) = x(x - 3)(x + 3)$$

Problem 25 At $x = 4$ the function $f(x) = x^4 - 6x^3 + 4x^2 - 13$ is

1. increasing

2. decreasing
3. stationary
4. none of the above

Problem 26 Let A and B be two events such that $P(A) = \frac{1}{5}$, while $P(A \text{ or } B) = \frac{1}{2}$. Let $P(B) = 'p'$. For what values of 'p' are A and B independent?

Problem 27 Life expectancy (in days) of electronic component has density function $p(x) = 1/x^2$ for $x \geq 1$, and $p(x) = 0$ for $x < 1$. Find the probability that, the component lasts between 0 and 2 day.

Problem 28 Let X be a discrete random variable with the following probability mass function. Find the Probability $P(0.25 < X < 0.75)$

$$P_x(x) = \begin{cases} 0.1 & \text{for } x = 0.2 \\ 0.2 & \text{for } x = 0.4 \\ 0.2 & \text{for } x = 0.5 \\ 0.3 & \text{for } x = 0.8 \\ 0.2 & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

Problem 29 Let X be a continuous random variable with probability density function as follows. Find $P[X < 1/2]$

$$\begin{cases} f_x(x) = x^2 + \frac{2}{3} & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Problem 30 When Ravi plays chess against his favourite computer program, he wins with probability 0.60, loses with probability 0.10, and 30% of the games results a draw. Assume independence. Find the probability that Ravi's first win happens when he plays his third game.

2 Test for the Economics track

Problem 31 Calculate the following limit

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - \sqrt{x^2 - 1})$$

Problem 32 Calculate the rank of the following matrix:

$$\begin{pmatrix} 5 & 3 & 0 \\ 1 & 2 & -4 \\ -2 & -4 & 8 \end{pmatrix}$$

Problem 33 For what value of x , the following matrix is singular?

$$\begin{pmatrix} 5 - x & x + 1 \\ 2 & 4 \end{pmatrix}$$

Problem 34 At which x does the function $f(x) = 2x^2 - \ln(x) - 2$ has minimum?

Problem 35 Life expectancy (in days) of electronic component has density function $p(x) = 1/x^2$ for $x \geq 1$, and $p(x) = 0$ for $x < 1$. Find the probability that, the component lasts between 0 and 2 day.

Problem 36 Let X be a discrete random variable with the following probability mass function. Find the Probability $P(0.25 < X < 0.75)$

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Problem 37 Let X be a continuous random variable with probability density function as follows. Find $P[X < 1/2]$

$$\begin{cases} f_x(x) = x^2 + \frac{2}{3} & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Problem 38 When Ravi plays chess against his favourite computer program, he wins with probability 0.60, loses with probability 0.10, and 30% of the games results a draw. Assume independence. Find the probability that Ravi's first win happens when he plays his third game.

Problem 39 Possible values of x are such that $x \in [0, 10]$. Consider the following two sets:

$$A = \{x : 0 \leq x \leq 5\}, B = \{x : 3 \leq x \leq 7\}.$$

Characterize the event that

$$A^c \cap B^c$$

Write down the upper limit of this new set.

Problem 40 Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. How much is $(A + 2B)^{-1}$? As your answer, write the sum of all the elements of the resulting matrix.

Problem 41 Let $z = x^2 \ln(y^3)$. Calculate the partial derivative $\frac{\partial z}{\partial y}$ at point $(x, y) = (2, 4)$

Problem 42 The following equation relates x and y .

$$\ln(1 + xy + y^2) + x = 0$$

Calculate the partial derivative $\frac{\partial y}{\partial x}$. Calculate its value at point $x = 2$

Problem 43 Calculate the integral

$$\int (3x^2 - 6x + 3) dx$$

at point $x = 2$

Problem 44 For which values of the parameter a the function

$$f(x, y) = -6x^2 + (2a + 4)xy - y^2 + 4ay$$

is strictly concave? Find the interval of such a . Then, as your answer, calculate the sum the upper and lower limit of a .

Problem 45 For which value of the parameter A the function

$$f(x) = \begin{cases} Ax^3 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

is a density function of a random variable x ?

Problem 46 Calculate the following integral

$$\int \frac{x^3}{x+5} dx$$

at point $x = -4$ (assuming that the constant of integration is zero)

Problem 47 Find the local minimum

$$f(x) = x^3 + y^3 - 3xy$$

As your answer give the x -coordinate

Problem 48 Calculate the derivative $y'(x)$ at point $x = -1$ if

$$2x^2 + 6xy + y^2 = 18$$

y can take only non-negative values.

Problem 49 Using the Lagrange multiplier method find the maximum of the following constrained maximization problem

$$\max 4x^2 + 3xy + 6y^2$$

subject to $x = 56 - y$.

Problem 50 For the following function

$$F(x, y) = (x^2y, xy^2)$$

compute the differential of the local inverse of F at the point $F(2, 1)$. As your answer, sum up all the elements of the resulting matrix.

Problem 51. Find the maximal value of the function

$$f(x, z) = 30x + 33z$$

subject to the system of inequalities

$$\begin{cases} x + 2z \leq 20 \\ x + z \leq 16 \\ 3x + 2z \leq 44 \\ x \geq 0, z \geq 0 \end{cases}$$

Problem 52. Find the limit (use only the real-valued root)

$$\lim_{x \rightarrow 1} \sqrt{1-x} \ln \ln 1/x$$

Problem 53. Given that a person has allergy, an allergy test gives a positive result (indication of allergy) with 80% probability. If a person who doesn't have allergy does the test, it gives a positive result (false positive) with 50% probability. It is known that around 20% of people can develop seasonal allergies.

Imagine you go to the doctor and you do the allergy test which gives a positive result. What is the probability that you have allergy (write only the numerical answer, without the % sign)?

Problem 54. Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 4 & 0 & 0 \\ -3 & 5 & 9 \\ 1 & 0 & 3 \end{bmatrix}$$

As your answer, write the sum of the eigenvalues.