# Dynamic Macroeconomic Models

4th module 2015/16

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# **Course Information**

The goal of this course is to equip the students with the basic tools necessary to understand the papers written in the modern macroeconomic tradition, namely, the dynamic programming as applied to macroeconomics. We will study both particular dynamic programming problems which allow for a closed form solution and the general way of approximately solving any macroeconomic problem that could be represented as a dynamic programming problem under rational expectations. The study of specific models will take a back seat to mastering the techniques.

More specific description of the course topics follow below.

1. This overview is intended for individual reading. The aim of this topic is to give the students a brief introduction into the evolution of macroeconomics as a science in the 2nd half of 20th century, as well as to introduce the common core and the areas of disagreement present in the model macroeconomic modeling.
2. This topic introduces the dynamic programming theory, first in a (generically) finite horizon version usually employed in engineering applications, and an infinite horizon formulation that is preferred in the economic literature. We will attempt to generate an intuition for the backward induction step of the dynamic programming algorithm, and see how translation to the infinite horizon leads naturally to the celebrated Bellman equation. The topic will close by discussing first order conditions a solution to the infinite horizon dynamic programming problem should satisfy.
3. This is the major topic which discusses methods of analytical and numerical solutions for dynamic programming problems that typically arise in macroeconomic applications. First, we will discuss the generally applicable to any dynamic programming problem solution method – the value function iteration, both as a numeric and an analytic algorithm. After demonstrating the power and generality of this approach, we will discuss its shortcomings, namely, the curse of dimensionality. Then we proceed to the policy iterations method, which is typically faster than the value function iterations but requires more stringent conditions on the problem for convergence to be assured.

Later, we move on to the special cases of the general dynamic programming problem. Log-linearization is a general method of transforming nonlinear first-order conditions and the laws of motion of a particular macroeconomic dynamic programming problem into a system of linear expectational difference equations. We then discuss two particular methods of solving such equations: the method of undetermined coefficients, generally applicable to the systems of low dimensionality, and the general Blanchard-Kahn method that is widely used to solve the systems of linear expectational difference equations under rational expectations for any dimensionality. We discuss both the advantages (no restrictions on the dimensionality) and shortcomings (only local solution available) of the Blanchard-Kahn method.

Finally, we discuss a specific form of linear-quadratic dynamic programming problem that is widely used in a particular subset of macroeconomics – monetary economics – that has received a lot of attention in the recent decades. The advantage of the linear-quadratic approach is in its ability to generate policy functions as closed form (even linear) solutions. We discuss the applicability of the certainty equivalence principle that is naturally illustrated using linear-quadratic problems.

# **Course Outline and Reading List**

# **Major Textbooks**

B D. Bertsekas: *Dynamic Programming and Optimal Control*, Athena Scientific, 2005.

LS Ljungquist, Lars, and Thomas J. Sargent: *Recursive Macroeconomic Theory, 2nd ed.*, Cambridge: MIT Press, 2004.

M George McCandles: *The ABCs of RBCs: An Introduction to Dynamic Macroeconomic Models.* Cambridge: Harvard University Press, 2008.

**Additional Textbooks and Reading**

AC Ada, Jerome and Russell Cooper. Dynamic Economics. MIT Press, 2003.

BF Blanchard, O. and S. Fisher: *Lectures on Macroeconomics*. MIT Press, 1989.

SL Stokey, Nancy L., Robert E. Lucas, Jr., and Edward C. Prescott: *Recursive Methods in Economic Dynamics*. Cambridge: Harvard University Press, 1989.

1. Overview of the Macroeconomics (for general reading).

* Blanchard, O., "What Do We Know About Macroeconomics that Fisher and Wicksell Did Not?" QJE, November 2000, 115:4, 1375-1410.
* Blanchard, O., “The State of Macro”, NBER WP 14259.
* Woodford, M., "Revolution and Evolution in Twentieth-Century Macroeconomics," Available at <http://www.columbia.edu/~mw2230/macro20C.pdf>.

1. Discrete Time Dynamic Programming: Finite and Infinite Horizon

* B Volume 1, Chapter 1.
* SL Chapters 1-4, LS Chapters 3-4.

##### Numerical Solution Methods

IIa. Value Function Iteration

* LS Chapter 4.

Applications:

Consumption and Savings – discrete time.

* M Chapter 3.

One-Sector Model of Economic Growth

* SL Chapter 5.1, 5.4, 5.7, LS Chapter 11.

IIb. Policy Function Iteration

* LS Chapter 4.

Application:

Search Model.

* LS Chapter 6.3.

IIc. Log-Linearization, Method of Undetermined Coefficients, Blanchard-Kahn

Application:

RBC Model

* M Chapters 1, 6.

IId. Linear-Quadratic Problem

* LS Chapter 5, M Chapter 7

Application:

* Monetary Policy

**Evaluation**

60% - final exam, 20% - midterm exam, 20% - homework and class participation.